Bianchi Type I Universes with dilaton and magnetic fields

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We consider the dynamics of a Bianchi type I space-time in the presence of dilaton and magnetic fields. The general solution of the Einstein-dilaton-Maxwell field equations can be obtained in an exact parametric form. Depending on the numerical values of the parameters of the model there are three distinct classes of solutions. The time evolution of the mean anisotropy, shear and deceleration parameters is considered in detail and it is shown that a magnetic-dilaton anisotropic Bianchi type I geometry does not isotropize, the initial anisotropy being present in the universe for all times.
PACS Numbers:98.80.Hw, 98.80.Bp, 04.20.Jb

I. INTRODUCTION

In a series of recent papers [1], [2], Bianchi type I cosmological models based on the field equations derived from the action

$$S=-\frac{1}{2}\int d^4x\sqrt{-g}\left[R-2\gamma\left(\nabla\phi\right)^2+e^{-2\alpha\phi}F^2\right], \tag{1}$$

where $\alpha = const. > 0$, $\gamma = const. > 0$ have been considered. In Eq. (1) ϕ is the dilaton field and F is the Maxwell two-form field, with components $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The action (1) can be obtained from the string frame low energy effective action

$$\hat{S}=-\int d^4x\sqrt{-\hat{g}}e^{-2\alpha\phi}\left[\hat{R}-2\gamma\left(\hat{\nabla}\phi\right)^2+F^2\right], \tag{2}$$

by means of the conformal transformation $\hat{g}_{\mu\nu} = e^{2\alpha\phi}g_{\mu\nu}$. As a particular case these models include Einstein-Maxwell ($\alpha = \gamma = 0$) and low energy string theory ($\alpha = 2$, $\gamma = 2$). In order to study the corresponding anisotropic cosmological model Salim, Sautu and Martins [1] wrote the field equations as an autonomous system of differential equations, with the time coordinate redefined in term of the comoving volume element. Several particular solutions of Kasner and exponential type have been considered and the exact general solution for the model has been obtained. The main conclusions of the authors of [1] are that the typical oscillatory dynamics of the model based on the Einstein-Maxwell theory in a Bianchi type I geometry is not present when approaching the initial singularity, due to the presence of the dilaton field and that the process of isotropization is completely absent. But as mentionned by Bannerjee and Ghosh [2], these solutions apparently do not satisfy all the field equations, the substitution of the solutions into the equation of motion of the dilaton field leading to some inconsistencies. In their investigation of the Bianchi type I cosmology Bannerjee and Ghosh [2] obtained a simple bi-axial (two equal scale factors) solution, with scale factors starting from an initial pointlike singularity of zero volume. Hence a general solution of the Bianchi type I Einstein-Maxwell-dilaton model based on the action (1) seems to be still missing.

The investigation of cosmological models with magnetic fields is definitely of major physical interest. The possibility of the existence of a homogeneous intergalactic field (possible of primordial origin) is not ruled out by observations. In an investigation of a magnetic Bianchi type I Universe Madsen [3] obtained a bound of the cosmological magnetic field given by $B \leq 3\times 10^{-4}h\sqrt{\delta\Omega}\left(1+z_d\right)^{-\frac{1}{2}}G$, where $\Omega = \frac{8\pi G\rho}{3H^2}$ is the cosmological density parameter, h is the Hubble parameter measured in units of $100kms^{-1}$, z_d is the redshift at which the anisotropy begins to grow and $\delta = 10^{-4}$. Although no magnetism has been seen yet on a cosmic scale or on a void's scale, magnetism has been detected in some superclusters of galaxies, clusters of galaxies, halos of elliptical galaxies, in the disk of spiral galaxies etc. [4].

From a theoretical point of view Einstein-Maxwell type magnetic models for the class of hypersurface-orthogonal Bianchi I cosmologies with a γ - law perfect fluid and a pure, homogeneous source-free magnetic field have been recently analyzed by using methods from the qualitative theory if dynamical systems in [5]. The main physical results

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of this investigation are that if $0 < \gamma \leq \frac{4}{3}$, magnetic fields do not prevent isotropization in the absence of spatial curvature. Even in the absence of anisotropic spatial curvature mixmaster-like oscillations also occur.

On the other hand in an attempt to address the potential inherited from string theory to eliminate the initial cosmological singularity, Gasperini and Veneziano [6] initiated a program known as the pre-big bang scenario. The field equations of the pre-big bang cosmology are based on the low energy effective action resulting from string theory. Pre-big bang cosmological models, in which there is no need to introduce inflation or to fine-tune potentials, have many attractive features. Inflation is natural, thanks to the duality symmetries of string cosmology and the initial condition problem is decoupled from the singularity problem. Finally, quantum instabilities (pair creation) is able to heat up an initially cold universe and generate a standard hot big bang with the additional features of homogeneity, flatness and isotropy (for a recent and extensive review of string cosmology see [7]). Exact anisotropic Bianchi type I cosmological solutions of the low energy string effective action in the presence of homogeneous magnetic fields have been obtained by Giovannini [8], [9]. The obtained cosmologies are fully anisotropic and only quadratic curvature corrections are able to isotropize the geometry of the Universe. The presence of magnetic seeds can induce further anisotropies, which can, in principle, be present also at late times.

It is the purpose of the present paper to consider the general solution of the Einstein-dilaton-Maxwell field equations derived from the action (1). The mathematical description of this system can be reduced to a single second-order differential equation and the general solution of the system can be represented in an exact parametric form. The behaviour of the basic cosmological parameters of observational interest like the anisotropy and deceleration parameters is also considered in detail. As a general result of our study we show that the dilaton-magnetic Bianchi type I geometry does not isotropize and hence it is very unlikely that the present day cosmological magnetic fields could be remnants from a pre-big bang era.

Throughout this paper we use the Landau-Lifshitz conventions [10]. The present paper is organized as follows. The field equations are written down in Section II. The general solution of the field equations is obtained in Section III. In Section IV we discuss and conclude our results.

II. FIELD EQUATIONS, GEOMETRY AND CONSEQUENCES

The field equations obtained from the variation of the action (1) are given by:

$$\left(e^{-2\alpha\phi}F^{\mu\nu}\right)_{;\mu} = 0, \tag{3}$$

$$\phi_{;\mu}^{;\mu} + \frac{\alpha}{2\gamma}e^{-2\alpha\phi}F^2 = 0, \tag{4}$$

$$R_{\mu\nu} = 2\gamma\phi_{;\mu}\phi_{;\nu} + 2e^{-2\alpha\phi}\left(F_{\mu\lambda}F_{\nu}^{\lambda} - \frac{1}{4}g_{\mu\nu}F^2\right), \tag{5}$$

$$\epsilon^{\alpha\lambda\mu\nu}\partial_\lambda F_{\mu\nu} = 0. \tag{6}$$

The last equation is the Bianchi identity for the Maxwell field. Throughout this paper we consider natural units with $c = 8\pi G = 1$. A semi-colon ; denotes the covariant derivative with respect to the metric.

The line element of the Bianchi type I geometry, generalizing to the anisotropic case the flat Robertson-Walker space-time is given by

$$ds^2 = dt^2 - a_1^2(t)dx^2 - a_2^2(t)dy^2 - a_3^2(t)dz^2. \tag{7}$$

For this metric we introduce the following variables [11]: the volume scale factor $V = \Pi_{i=1}^3 a_i$, the directional Hubble factors $H_i = \frac{\dot{a}_i}{a_i}, i = 1,2,3$, the mean Hubble factor $H = \frac{1}{3}\sum_{i=1}^3 H_i$ and we denote $\Delta H_i = H - H_i, i = 1,2,3$. From these definitions we immediately obtain $H = \frac{1}{3}\frac{\dot{V}}{V}$.

Without any loss of generality we assume that the magnetic field is aligned along the x - direction. Its unique component is $F_{23} = F_0 = const.$ and it satisfies both Maxwell equations (3) and (6). Then $F^2 = F_{\mu\nu}F^{\mu\nu} = 2\frac{F_0^2}{V^2}a_1^2$.

We also introduce the symbol $\varepsilon_i, i = 1,2,3$ defined by $\varepsilon_1 = -1, \varepsilon_2 = \varepsilon_3 = +1$. ε_i has the obvious property $\sum_{i=1}^3 \varepsilon_i = +1$.

With the use of the variables introduced above the non-trivial field equations (3)-(6) are given by

$$3\dot{H}+\sum_{i=1}^{3}H_i^2=-2\gamma\dot{\phi}^2-F_0^2\frac{a_1^2}{V^2}e^{-2\alpha\phi}, \tag{8}$$

$$\frac{1}{V}\frac{d}{dt}\left(VH_i\right)=\varepsilon_i F_0^2\frac{a_1^2}{V^2}e^{-2\alpha\phi}, i=1,2,3, \tag{9}$$

$$\frac{1}{V}\frac{d}{dt}\left(V\dot{\phi}\right)=\frac{\alpha}{2\gamma}F_0^2\frac{a_1^2}{V^2}e^{-2\alpha\phi}. \tag{10}$$

Adding Eqs.(9) one obtain

$$\frac{1}{V}\frac{d}{dt}\left(3VH\right)=F_0^2\frac{a_1^2}{V^2}e^{-2\alpha\phi}. \tag{11}$$

Substituting the term $\frac{F^2}{2}e^{-2\alpha\phi}$ expressed from Eq.(11) into Eq.(10) we find

$$\frac{1}{V}\frac{d}{dt}\left(V\dot{\phi}\right)=\frac{\alpha}{2\gamma}\frac{1}{V}\frac{d}{dt}\left(3VH\right), \tag{12}$$

or, equivalently,

$$\dot{\phi}=\frac{\alpha}{2\gamma}\frac{\dot{V}}{V}+\frac{\alpha}{2\gamma}\frac{m}{V}, \tag{13}$$

with m an arbitrary constant of integration. Integrating Eq.(13) we obtain for the dilaton field

$$\phi=\phi_0+\frac{\alpha}{2\gamma}\ln V+\frac{\alpha m}{2\gamma}\int\frac{dt}{V}, \tag{14}$$

where ϕ_0 is a constant of integration.

We combine now the dilaton field Eq.(10) and the gravitational field equations (9) to find

$$\frac{1}{V}\frac{d}{dt}\left(VH_i\right)=\varepsilon_i\frac{2\gamma}{\alpha}\frac{1}{V}\frac{d}{dt}\left(V\dot{\phi}\right), i=1,2,3, \tag{15}$$

which gives

$$H_i=\varepsilon_i\frac{2\gamma}{\alpha}\dot{\phi}+\frac{K_i}{V}, i=1,2,3. \tag{16}$$

$K_i, i=1,2,3$ are arbitrary constants of integration. With the use of Eq. (13) we can express Eq.(16) in the equivalent form

$$H_i=\varepsilon_i\frac{\dot{V}}{V}+\frac{K_i+m\varepsilon_i}{V}, i=1,2,3. \tag{17}$$

From Eqs.(17) it follows that the integration constants K_i must satisfy the consistency condition

$$m+\sum_{i=1}^{3}K_i=0. \tag{18}$$

The scale factors are obtained by integrating Eq. (17) and are given by

$$a_i=a_{i0}V^{\varepsilon_i}e^{(K_i+m\varepsilon_i)\int\frac{dt}{V}}, i=1,2,3, \tag{19}$$

with $a_{i0}, i=1,2,3$ non-negative constants of integration.

We substitute now the dilaton field Eq.(13), $H_i, i=1,2,3$ given by Eqs.(17) and the magnetic field from Eq. (11) into the field equation Eq.(8) to obtain the basic equation describing the evolution of the dilaton and magnetic field filled Bianchi type I space-time:

$$\frac{\ddot{V}}{V}+C\frac{\dot{V}^2}{V^2}+B\frac{\dot{V}}{V^2}+\frac{K}{V^2}=0, \tag{20}$$

where $C=1+\frac{\alpha^2}{4\gamma}$, $B=\sum_{i=1}^{3}\left(\varepsilon_i K_i+m\right)+\frac{\alpha^2}{2\gamma}m$ and $K=\frac{\sum_{i=1}^{3}(K_i+m\varepsilon_i)^2}{2}+\frac{\alpha^2}{4\gamma}m^2$.

Denoting $\dot{V}=u$, Eq.(20) is transformed into

$$\frac{dV}{V}=-\frac{udu}{Cu^2+Bu+K}. \tag{21}$$

The general solution of Eq.(21) can be formally represented in the form

$$V=V_0e^{-\int\frac{udu}{Cu^2+Bu+K}}, \tag{22}$$

with $V_0>0$ a constant of integration.

The physical quantities of observational interest in cosmology are the expansion scalar $\theta=3H$, the mean anisotropy parameter A, the shear scalar Σ^2 and the deceleration parameter q defined according to

$$A=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\Delta H_i}{H}\right)^2, \tag{23}$$

$$\Sigma^2=\frac{1}{2}\left(\sum_{i=1}^{3}H_i^2-3H^2\right)=\frac{3}{2}AH^2, \tag{24}$$

$$q=\frac{d}{dt}\frac{1}{H}-1. \tag{25}$$

With the use of Eqs.(17) we can express the mean anisotropy and the deceleration parameter in the alternative form

$$A=8+\frac{2\sum_{i=1}^{3}\left(\varepsilon_i K_i+m\right)}{VH}+\frac{1}{3}\frac{\sum_{i=1}^{3}\left(K_i+m\varepsilon_i\right)^2}{V^2H^2}, \tag{26}$$

$$\Sigma^2=H^2\left(12+\frac{3\sum_{i=1}^{3}\left(\varepsilon_i K_i+m\right)}{VH}+\frac{1}{2}\frac{\sum_{i=1}^{3}\left(K_i+m\varepsilon_i\right)^2}{V^2H^2}\right). \tag{27}$$

The sign of the deceleration parameter indicates whether the cosmological model inflates. The positive sign corresponds to standard decelerating models while a negative sign indicates inflation. The deceleration parameter can be expressed as a function of the variable u in the general form

$$q=2+\frac{3\left(Cu^2+Bu+K\right)}{u^2}. \tag{28}$$

In the isotropic limit the mean anisotropy parameter is zero.

III. GENERAL SOLUTION OF THE FIELD EQUATIONS

In the previous Section we have obtained the basic equations describing the dynamics of an anisotropic Bianchi type I universe filled with dilaton and magnetic fields. The field equations of the model can be reduced to a single equation (20) describing the evolution of the anisotropic universe. According to the sign of the parameter $\Delta=B^2-4CK$ the second order differential equation Eq.(20) has three distinct classes of solutions.

For $\Delta=B^2-4CK>0$ the general solution of Eq. (20) is

$$V=V_0\left(u-u_+\right)^{m_+}\left(u-u_-\right)^{m_-}, \tag{29}$$

where we denoted $u_\pm = \frac{-B\pm\sqrt{\Delta}}{2C}$ and $m_\pm = \mp\frac{u_\pm}{\sqrt{\Delta}}$. Therefore the general solution of the field equations can be expressed in the following parametric form, with u taken as parameter:

$$t - t_0 = -\frac{V_0}{C}\int (u-u_+)^{m_+-1}(u-u_-)^{m_--1}\,du, \tag{30}$$

$$H(u) = \frac{1}{3V_0}\frac{u}{(u-u_+)^{m_+}(u-u_-)^{m_-}}, \tag{31}$$

$$a_i(u) = a_{i0}(u-u_+)^{p_i^+}(u-u_-)^{p_i^-}, i = 1,2,3, \tag{32}$$

$$A(u) = 8 + \frac{6\sum_{i=1}^3(\varepsilon_i K_i + m)}{u} + \frac{3\sum_{i=1}^3(K_i + m\varepsilon_i)^2}{u^2}, \tag{33}$$

$$\Sigma^2(u) = \frac{1}{9V_0^2}\frac{u^2}{(u-u_+)^{2m_+}(u-u_-)^{2m_-}}\left(12 + \frac{9\sum_{i=1}^3(\varepsilon_i K_i + m)}{u} + \frac{9}{2}\frac{\sum_{i=1}^3(K_i + m\varepsilon_i)^2}{u^2}\right), \tag{34}$$

$$q(u) = 2 + \frac{3C(u-u_+)(u-u_-)}{u^2}, \tag{35}$$

$$\phi(u) = \phi_0 + \frac{\alpha}{2\gamma}\ln\left[(u-u_+)^{n_+}(u-u_-)^{n_-}\right], \tag{36}$$

where we denoted $p_i^\pm = \varepsilon_i m_\pm \mp \frac{K_i + m\varepsilon_i}{\sqrt{\Delta}}$, $i = 1,2,3$ and $n_\pm = m_\pm \mp \frac{m}{\sqrt{\Delta}}$. For $u_\pm > 0$ the condition of the reality of the physical parameters implies that this solution is defined only for values of the parameter u so that $u > u_+, u_-$. For $u_\pm < 0$ there are no restrictions on the allowed range of the parameter u.

For $\Delta = B^2 - 4CK = 0$ the general solution of the field equations is given by

$$t - t_0 = -\frac{V_0}{C}\int \frac{e^{\frac{m_0}{u-u_0}}}{(u-u_0)^{\frac{1}{C}+2}}du, \tag{37}$$

$$V(u) = V_0(u-u_0)^{-\frac{1}{C}}e^{\frac{m_0}{u-u_0}}, \tag{38}$$

$$H(u) = \frac{1}{3V_0}u(u-u_0)^{\frac{1}{C}}e^{-\frac{m_0}{u-u_0}}, \tag{39}$$

$$a_i(u) = a_{i0}(u-u_0)^{-\frac{\varepsilon_i}{C}}e^{\frac{1}{C}\frac{(m+Cm_0)\varepsilon_i + K_i}{u-u_0}}, i = 1,2,3, \tag{40}$$

$$A(u) = 8 + \frac{6\sum_{i=1}^3(\varepsilon_i K_i + m)}{u} + \frac{3\sum_{i=1}^3(K_i + m\varepsilon_i)^2}{u^2}, \tag{41}$$

$$\Sigma^2(u) = \frac{1}{9V_0^2}u^2(u-u_0)^{\frac{2}{C}}e^{-\frac{2m_0}{u-u_0}}\left(12 + \frac{9\sum_{i=1}^3(\varepsilon_i K_i + m)}{u} + \frac{9}{2}\frac{\sum_{i=1}^3(K_i + m\varepsilon_i)^2}{u^2}\right), \tag{42}$$

$$q(u) = 2 + \frac{3C(u-u_0)^2}{u^2}, \tag{43}$$

$$\phi(u)=\phi_0+\frac{\alpha}{2\gamma C}\left[\frac{u_0+m}{u-u_0}-\ln\left(u-u_0\right)\right], \tag{44}$$

where we denoted $u_0=-\frac{B}{2C}$ and $m_0=\frac{u_0}{C}$. In order to have well-defined (real) physical quantities it is necessary that the parameter u satisfies the condition $u>u_0$.

For $\Delta=B^2-4CK<0$ the general solution of the field equations is

$$t-t_0=-\frac{V_0}{C}\int\frac{\exp\left[\frac{B}{2C^2\Delta_0}\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right)\right]}{\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]^{1+\frac{1}{2C}}}du, \tag{45}$$

$$V(u)=V_0\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]^{-\frac{1}{2C}}\exp\left[\frac{B}{2C^2\Delta_0}\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right)\right], \tag{46}$$

$$H(u)=\frac{1}{3V_0}u\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]^{\frac{1}{2C}}\exp\left[-\frac{B}{2C^2\Delta_0}\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right)\right], \tag{47}$$

$$a_i(u)=a_{i0}\frac{\exp\left\{\frac{1}{C\Delta_0}\left[\varepsilon_i\left(\frac{B}{2C}-m\right)-K_i\right]\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right)\right\}}{\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]^{\frac{\varepsilon_i}{2C}}}, i=1,2,3, \tag{48}$$

$$A(u)=8+\frac{6\sum_{i=1}^3\left(\varepsilon_iK_i+m\right)}{u}+\frac{3\sum_{i=1}^3\left(K_i+m\varepsilon_i\right)^2}{u^2}, \tag{49}$$

$$\Sigma^2(u)=\frac{1}{9V_0^2}\frac{u^2\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]^{\frac{1}{C}}\left(12+\frac{9\sum_{i=1}^3(\varepsilon_iK_i+m)}{u}+\frac{9}{2}\frac{\sum_{i=1}^3(K_i+m\varepsilon_i)^2}{u^2}\right)}{\exp\left[\frac{B}{C^2\Delta_0}\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right)\right]}, \tag{50}$$

$$q(u)=2+3C\frac{\left(u+\frac{B}{2C}\right)^2+\Delta_0^2}{u^2}, \tag{51}$$

$$\phi(u)=\phi_0-\frac{\alpha}{4C\gamma}\ln\left[\left(u+\frac{B}{2C}\right)^2+\Delta_0^2\right]+\frac{\alpha}{2\gamma C\Delta_0}\left(\frac{B}{2C}-m\right)\arctan\left(\frac{u+\frac{B}{2C}}{\Delta_0}\right), \tag{52}$$

where $\Delta_0=\frac{\sqrt{-\Delta}}{2C}$.

IV. DISCUSSIONS AND FINAL REMARKS

In order to consider the general effects of the magnetic field on the dynamics and evolution of the Bianchi type I space-time we also present the solutions corresponding to the pure anisotropic dilatonic universe, without magnetic field, $F_0=0$. These solutions are not new, they have already been discussed in [12] and [13]. In the absence of the magnetic field the dilatonic universe is described by

$$V=V_0t, H=\frac{1}{3t}, a=a_{i0}t^{p_i}, \tag{53}$$

$$A=\frac{3K^2}{V_0^2}=const., q=2=const., \phi=\phi_0\ln t, \tag{54}$$

where $\phi_0 = \phi_0' \sqrt{\frac{1}{4\gamma}\left(\frac{2}{3} - \frac{K^2}{V_0^2}\right)}$, with ϕ_0' an arbitrary constant of integration. $K_i, i = 1, 2, 3$ are constants of integration satisfying the condition $\sum_{i=1}^{3} K_i = 0$. We also denoted $K^2 = \sum_{i=1}^{3} K_i^2$. The coefficients $p_i = \frac{1}{3} + \frac{K_i}{V_0}, i = 1, 2, 3$ satisfy the relations $\sum_{i=1}^{3} p_i = 1$ and $\sum_{i=1}^{3} p_i^2 = \frac{1}{3} + \frac{K^2}{V_0^2}$. The geometry of the dilaton-field filled universe is of Kasner type. A universe of this type will never experience a transition to an isotropic phase and its evolution is non-inflationary for all times.

The variation of the volume scale factor V of the magnetic and dilaton fields filled Bianchi type I space-time is represented, for all three models, in Fig. 1.

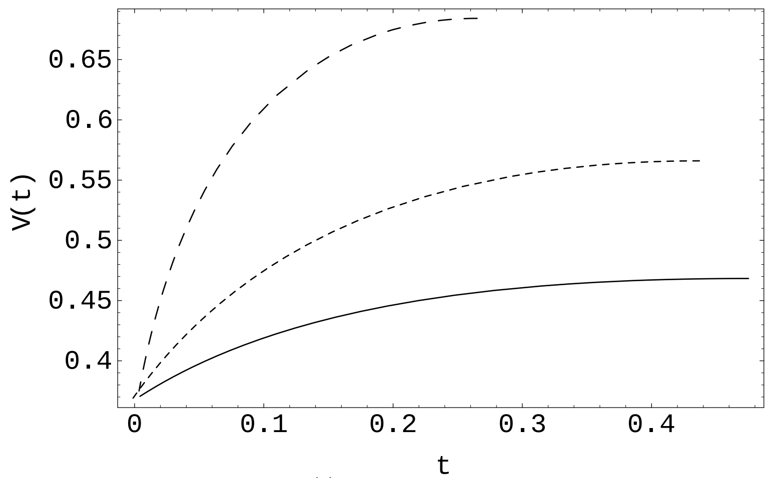

FIG. 1. Time evolution of the volume scale factor $V(t)$ of the magnetic-dilaton Bianchi I space-time for different values of the parameters: $B = 4$, $C = 1$ and $K = 1$ $(\Delta > 0)$ (solid curve), $B = 1$, $C = 1$, $m = 2.5$ and $K = \frac{1}{4}$ $(\Delta = 0)$ (dotted curve) and $B = 2$, $C = 2$ and $K = 2$ $(\Delta < 0)$ (dashed curve).

The evolution of the universe is generally expansionary. The singularity behaviour depends on the sign of the constants $u_{\pm}$ and u_0, the real rooots of the equation $Cu^2 + Bu + K = 0$. If all these roots are positive, then a singular behaviour of the scale factors and of the volume scale factor cannot be avoided, with the singularity corresponding to values $u = u_{\pm}$ and $u = u_0$ of the parameter. In this case the range of the parameter u is restricted to $u \geq u_{\pm}$ and $u \geq u_0$. In the case of complex $u_{\pm}$ $(\Delta < 0$), there is no singular point in the time evolution of the scale factors $a_i, i = 1, 2, 3$. The asymptotic behaviour of the solution in the limit of large $\dot{V} = u$ (that is, for a very rapid expansion of the volume of the universe) can be obtained from equation (21), by taking the limit $u \to \infty$. Therefore we obtain $V \sim u^{-\frac{1}{C}}$ and $V \sim t^{\frac{1}{1+C}}$, respectively. The scale factors are given by $a_i = a_{i0} t^{\varepsilon_i \frac{1}{1+C}} \exp\left[\frac{1+C}{C}\left(K_i + m\varepsilon_i\right) t^{\frac{C}{1+C}}\right], i = 1, 2, 3$. Since generally $C > 0$, there is no isotropic limit for a_i. In the case of small u (very slow expansion of the universe) from Eq. (21) we obtain $V \sim e^{-\frac{u}{B}} \left(K + Bu\right)^{\frac{K}{B^2}}$, but in this limiting case the solution of the field equations cannot be expressed in an analytical form.

The evolution of the mean Hubble factor of the universe is represented in Fig. 2.

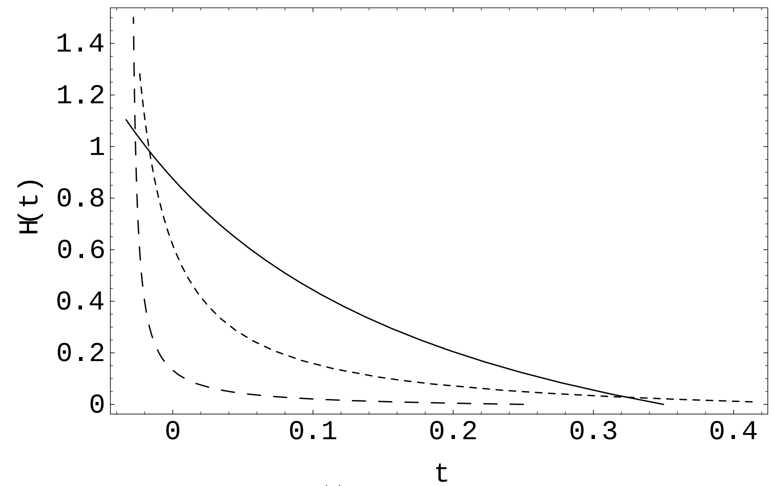

FIG. 2. Time evolution of the mean Hubble factor $H(t)$ of the magnetic-dilaton Bianchi I space-time for different values of the parameters: $B=4$, $C=1$ and $K=1$ $(\Delta>0)$ (solid curve), $B=1$, $C=1$, $m=2.5$ and $K=\frac{1}{4}$ $(\Delta=0)$ (dotted curve) and $B=2$, $C=2$ and $K=2$ $(\Delta<0)$ (dashed curve).

For all three models $H(t)$ is generally a decreasing function of time, with $H\rightarrow\infty$ for $t\rightarrow 0$. In the limit of large u we obtain $H\sim\frac{1}{t}$.

The dynamics of the mean anisotropy factor, presented in Fig. 3, shows a rapid time increase of the anisotropy of the dilaton-magnetic field filled universe.

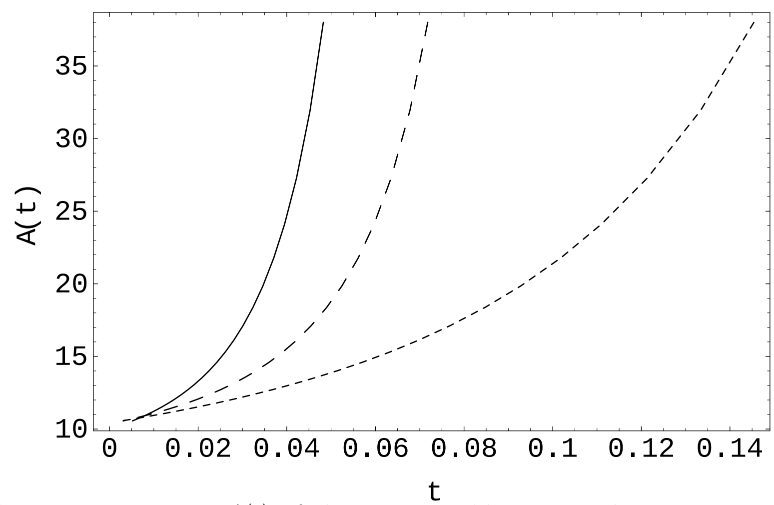

FIG. 3. Dynamics of the mean anisotropy $A(t)$ of the magnetic-dilaton Bianchi I space-time for different values of the parameters: $B=4$, $C=1$ and $K=1$ $(\Delta>0)$ (solid curve), $B=1$, $C=1$, $m=2.5$ and $K=\frac{1}{4}$ $(\Delta=0)$ (dotted curve) and $B=2$, $C=2$ and $K=2$ $(\Delta<0)$ (dashed curve). The integration constants $K_i, i=1,2,3$ have been normalized so that $6\sum_{i=1}^{3}\left(\varepsilon_iK_i+m\right)=1$ and $3\sum_{i=1}^{3}\left(K_i+m\varepsilon_i\right)^2=1$.

In the limit of large u we have $u=v_0t^{-\frac{1}{1+C}}, v_0=const.$ and the mean anisotropy parameter behaves as

$$A=8+2v_0\sum_{i=1}^{3}\left(\varepsilon_iK_i+m\right)t^{\frac{C}{1+C}}+3v_0^2\sum_{i=1}^{3}\left(K_i+m\varepsilon_i\right)^2t^{\frac{2C}{1+C}}. \tag{55}$$

Therefore the inclusion of a magnetic field will increase the anisotropy of the Bianchi type I space-time.

The dilaton field, presented in Fig. 4, is an increasing function of time. In the large u limit it behaves like $\phi\sim\phi_0+\frac{\alpha}{2\gamma}\frac{1}{1+C}\ln t+\frac{\alpha m(1+C)}{2\gamma C}t^{\frac{C}{1+C}}$. The dynamics of the universe is non-inflationary, with the deceleration parameter $q=2+3C$ for $u\rightarrow\infty$.

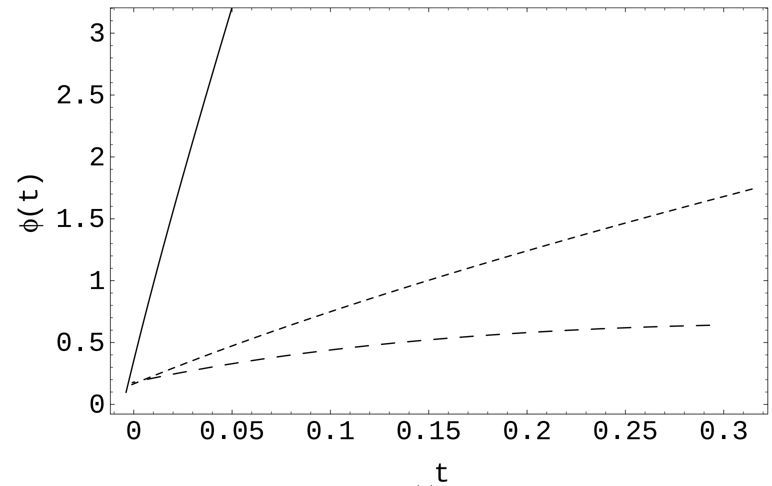

FIG. 4. Variation as a function of time of the dilaton field $\phi(t)$ of the magnetic-dilaton Bianchi I space-time for different values of the parameters: $B=4$, $C=1$ and $K=1$ $(\Delta>0)$ (solid curve), $B=1$, $C=1$, $m=2.5$ and $K=\frac{1}{4}$ $(\Delta=0)$ (dotted curve) and $B=2$, $C=2$, $K=2$ and $m=0.0675$ $(\Delta<0)$ (dashed curve). The constants α, γ and m have been normalized so that $\frac{\alpha}{4C\gamma}=1$ and $\frac{\alpha}{2\gamma C\Delta_0}\left(\frac{B}{2C}-m\right)=1$.

Finally we shortly discuss the behaviour of the physically important parameters of the solution in the string frame.

By assuming an anisotropic Bianchi type I geometry with line element $d\hat{s}^2=d\hat{t}^2-\sum_{i=1}^{3}\hat{a}_i^2\left(\hat{t}\right)\left(dx^i\right)^2$, with the string -frame time coordinate defined according to $\hat{t}=\int\exp\left(\alpha\phi\right)dt$, the string frame volume scale factor $\hat{V}$, the directional Hubble factors $\hat{H}_i, i=1,2,3$ and the mean Hubble factor $\hat{H}$ are related by means of the general transformations [12]

$$\hat{V}=Ve^{3\alpha\phi},\hat{H}_i=\left(H_i+\alpha\dot{\phi}\right)e^{-\alpha\phi},i=1,2,3,\hat{H}=\left(H+\alpha\dot{\phi}\right)e^{-\alpha\phi}. \tag{56}$$

In the string frame the mean anisotropy parameter is given by

$$\hat{A}=\frac{1}{3}\sum_{i=1}^{3}\left(\frac{\hat{H}_i-\hat{H}}{\hat{H}}\right)^2=\frac{A}{\left(1+\alpha\frac{\dot{\phi}}{H}\right)^2}. \tag{57}$$

With the use of Eq.(13) we obtain $\frac{\dot{\phi}}{H}=\frac{3\alpha}{2\gamma}+\frac{\alpha m}{2\gamma}\frac{1}{HV}=\frac{3\alpha}{2\gamma}+\frac{3\alpha m}{2\gamma}\frac{1}{u}$. In the limit of small u we obtain $\hat{A}\to 0$, while for large u we have $\hat{A}\sim A$. The conformal transformation factor $e^{\alpha\phi}$ is given, with the use of Eq. (14), by $e^{\alpha\phi}=\Phi_0V^{\frac{\alpha^2}{2\gamma}}\exp\left[\frac{\alpha^2m}{2\gamma}\int\frac{dt}{V}\right]$, where $\Phi=e^{\alpha\phi_0}=const.$. Therefore for the string frame scale factors we obtain

$$\hat{a}_i=\hat{a}_{i0}V^{\varepsilon_i+\frac{\alpha^2}{2\gamma}}\exp\left[\left(K_i+m\varepsilon_i+\frac{\alpha^2m}{2\gamma}\right)\int\frac{dt}{V}\right],i=1,2,3. \tag{58}$$

The mathematical form of the string frame scale factors is very similar to the form of scale factors in the Einstein frame. Therefore the dynamics of the dilaton-magnetic Bianchi type I universe has the same general features in both frames.

In the present paper we have considered the evolution of a dilaton and magnetic field filled Bianchi type I space-time. The general solution of the field equations can be expressed in an exact parametric form and, depending on the numerical values of some constants, three classes of solutions can be obtained. These solutions describe expanding universes, with non-inflationary Einstein frame evolutions. In the Einstein frame the Bianchi type I geometry does not isotropize and there is no Friedmann-Robertson-Walker limit of these cosmologies. In these models the dilaton field cannot provide a physical mechanism able to wash out the initial anisotropies. But such mechanism can be obtained by considering a matter component in the energy momentum tensor, a cosmological constant or some quadratic terms in the Lagrangian [8], [9], [12], [13]. Therefore it is very unlikely that simple pre-big bang models including only dilaton and magnetic fields can provide a realistic explanation of the presence of cosmological magnetic fields as remnants of a dilaton-magnetic era.

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